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Very-low-frequency magnetic plasma

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Abstract
We show that a set of current-carrying wires can exhibit an effective magnetic permeability at very low frequencies of a few hertz. The resonant permeability, which is negative above the resonance frequency, arises from the oscillations of the wires driven by the applied magnetic field. We show that a large, frequency-specific and tunable effective permeability can be realized for a wide range of strengths of the applied field.

1. Introduction

In this paper we give a recipe for a micro-structured material that behaves like a plasma of magnetic poles with an extremely low magnetic plasma frequency of a few hertz. The structure comprises two lattices of conducting wires carrying oppositely directed currents.

The properties of a free assembly of electrical charges are well known. The long-range Coulomb interaction and the finite mass of the charges combine to give an effective electrical permittivity of

$$\epsilon(\omega) = 1 - \frac{\omega_p^2}{\omega(\omega + i\gamma)} \tag{1}$$

where

$$\omega_p^2 = \frac{ne^2}{\epsilon_0 m} \tag{2}$$

and γ represents dissipation produced by electrical resistance. In good conductors, γ is small compared to ω_p.

This form of ϵ, and particularly the negative value of ϵ below the plasma frequency, give a plasma some unique properties. A magnetic plasma we define as having an analogous magnetic permeability, μ, which takes negative values in a range of frequencies and vanishes at some magnetic plasma frequency:

$$\omega = \omega_{mp}. \tag{3}$$

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Negative values of ϵ and/or μ impart some remarkable properties to a material. In particular, interfaces between two materials, one with $\epsilon < 0$ and the other with $\epsilon > 0$, will in general support surface modes. The structure of these modes is unrestricted by the free space wavelength of radiation and can sometimes result in extremely localized concentrations of energy within a few nanometres or less. These localized resonances are responsible for the strong absorbing powers of colloidal silver, and for the surface-enhanced Raman resonance effects also observed in these systems. Analogous effects can be expected when $\mu < 0$, though so far they remain theoretical predictions.

At lower frequencies the interest of 'negative epsilon' structures has recently been appreciated. Perhaps the most striking 'negative epsilon' property is to be found in a superconductor where in equation (1) $\gamma = 0$ and the permittivity is negative and diverges at zero frequency. This divergence is responsible for the expulsion of all electromagnetic fields from the superconductor as we can see by calculating the complex wavevector in this limit:

$$k = \frac{\omega\sqrt{\epsilon}}{c_0} \simeq \mathrm{i}\frac{\omega_p}{c_0} = \frac{\mathrm{i}}{c_0}\sqrt{\frac{ne^2}{\epsilon_0 m}}. \tag{4}$$

This tells us the decay of the fields in the superconductor and hence the London penetration depth:

$$\lambda = \frac{1}{-2\mathrm{i}k} = \sqrt{\frac{m}{4\mu_0 ne^2}}. \tag{5}$$

It is sometimes erroneously stated that a superconductor is a 'perfect diamagnet' with $\mu = 0$, whereas it is in fact a perfect plasma. Hence electrical plasmas can be seen to be interesting and unusual objects at all frequencies from DC to the UV.

By analogy we expect a magnetic plasma to be equally interesting, but with the role of electric and magnetic properties exchanged. For example whereas a superconductor expels all electromagnetic fields and requires that at its surface all magnetic fields are parallel and all electric fields are perpendicular, a magnetic plasma with $\mu \ll 0$ on the contrary will require magnetic fields to be perpendicular to its surface.

Producing a material with both ϵ and μ negative results in a negative refractive index. This requires a detailed consideration of matching conditions at the interface with vacuum, but it does prove to be a correct description of such systems. Veselago commented some time ago on the remarkable focusing property of a slab of such a material and Smith *et al* have shown that negative refractive index materials can be fabricated [1, 2]. More recently it has been shown that under the ideal conditions:

$$\epsilon = -1, \qquad \mu = -1, \tag{6}$$

a slab of material is in fact a perfect lens, not only bringing all propagating beams to a focus, but also restoring the amplitude and phase of the evanescent waves at the same point [3]. Normally the later are irretrievably lost to the image and hence the sharpness limited.

In fact, if we are concerned entirely with length scales that are much less that the wavelength of radiation, the electric and magnetic fields behave as separate entities. If we wish to focus only on the electrical component of an image, we only need to have that the value of the permittivity tensor perpendicular to the surface, $\epsilon_\perp = -1$, and the value of μ is irrelevant. Likewise the magnetic component only requires $\mu_\perp = -1$.

We can produce a material with a negative ϵ at almost any frequency from DC to the UV. Negative μ has been demonstrated in the gigahertz region and the technology is good well into megahertz frequencies. At lower frequencies the material suggested in this paper fills the gap.

2. A magnetic plasma

Although Maxwell's equations are symmetric with respect to magnetic and electric fields, Nature is less obliging. Only free electrical charges are available and magnetic monopoles have yet to be observed. Therefore constructing a magnetic plasma by direct analogy is not possible. In an earlier paper a trick was suggested for giving the appearance of a magnetic plasma: thin sheets of insulated conductor are wound around the surface of a cylinder so that when the cylinder is placed in a magnetic field currents flow in the conductor which magnetize the cylinder [4]. To an observer outside the cylinder this has the appearance of magnetic poles being propelled to opposite ends of the cylinder. An array of such cylinders was predicted to behave like a magnetic plasma, with a magnetic plasma frequency in the gigahertz range of frequencies. Such an approach to creating a magnetic plasma essentially relies on a resonant overscreening response to the magnetic field and can in principle be extended to submicron structures and infrared frequencies [5]. An alternative approach is to use insulating particles and to excite a resonant Mie mode of the required symmetry giving a bulk magnetization of the sample. Because of the sub-wavelength requirement for the inclusions, this method is practical at frequencies only where large dielectric constants are available [6].

We report here a completely different recipe for realizing a magnetic plasma at very low frequencies of a few hertz. We observe that current elements, just like charges, obey a long-range Coulomb interaction and are themselves capable of creating fields. However, unlike electrical charges, current elements respond to magnetic fields, and in the case of currents it is unlike currents that repel. Could we not have the elements of a magnetic plasma in a system of current-carrying wires?

The proposed structure consists of an array of long conducting wires parallel to the z-axis, pairs of which carry oppositely directed currents, $\pm I$, which we assume are driven by an infinite impedance source. When a magnetic field, $B\mathrm{e}^{-\mathrm{i}\omega t}$, is applied parallel to the y-axis (i.e. perpendicular to the wires), each wire carrying a positive current is subject to a force acting parallel to the x-axis of $-IB\mathrm{e}^{-\mathrm{i}\omega t}$ per unit length. The field experienced by each wire is made up of the externally applied field plus any further field due to its neighbours. The magnetic response of the structure has its origin in the resulting displacement of each wire from its equilibrium position which produces a net surface current density on the opposing yz-faces of the structure, which in turn induce a magnetic field, inside of the structure and also directed perpendicular to the wires. The magnitude and direction of this induced field determines the permeability of the structure in the effective medium limit. The long-range nature of the Coulomb force provides for the uniform restoring force between charged particles necessary for plasma oscillations in ionic plasmas. In the magnetic analogue there are two simple regimes. If the displacement of the wires is large relative to their spacing we can ignore local field corrections as averaging to zero and the wires experience the average induced field produced by surplus currents on the boundaries. The long-ranged force between current elements then acts to soften the vibrations of the wires under the tension exerted. By contrast in the low-field case, when the displacements are small compared to the interwire spacing, the wire remains trapped in the local field minimum which stabilizes the static structure.

We first treat the case where the applied field is large but assume that u_x, the displacement of the wires, remains small in comparison to their length. The equation of motion is written as

$$m\ddot{u}_x = -\alpha u_x - 2m\gamma\dot{u}_x - IB\exp(-\mathrm{i}\omega t), \tag{7}$$

where m is the mass per unit length of the wires and B is the amplitude of the average total field experienced by the wires. Here $-\alpha u_x$ is a restoring force due to the tension in the wires. The rate of damping is determined by the factor γ which results in a decay of free vibrations of the wires as $\exp(-\gamma t)$. The main sources of this loss for the systems we have in mind will be losses to the air or any fluid surrounding the structure and dissipation of energy at the points where the wires are supported.

Solving for u_x gives

$$u_x = \frac{I}{m\omega^2 - \alpha + 2\mathrm{i}m\gamma\omega} B\exp(-\mathrm{i}\omega t) = \frac{Im^{-1}}{\omega^2 - \omega_0^2 + 2\mathrm{i}\gamma\omega} B\exp(-\mathrm{i}\omega t), \tag{8}$$

where the wire resonance frequency, $\omega_0 = \sqrt{\alpha/m}$, was introduced.

Now we wish to calculate how this displacement of the wires magnetizes the structure. First we assume that the wires with positive current are arranged in a square array with lattice constant a. The wires with negative current are similarly arranged and their lattice centres the positive lattice though in fact in the large-field case the details of the wire arrangements are unimportant. The current density is $\pm Ia^{-2}$ per unit area. Hence we calculate that the upper yz-face of the structure acquires a surplus current density of $2Ia^{-2}u_x$, where the factor of two arises because each current lattice contributes equally to the surplus. There is a similar current deficit on the lower surface. Together the two sets of currents form a large solenoid inside which there is an induced magnetic field parallel to the y-axis which we calculate to be

$$B_{ind}\exp(-\mathrm{i}\omega t) = -2\mu_0 Ia^{-2}u_x = \frac{-2\mu_0 I^2 m^{-1}a^{-2}}{\omega^2 - \omega_0^2 + 2\mathrm{i}\gamma\omega} B\exp(-\mathrm{i}\omega t). \tag{9}$$

It remains to identify

$$B = B_{app} + B_{ind} \tag{10}$$

such that

$$B = B_{app}\left[1 + \frac{2\mu_0 I^2 m^{-1}a^{-2}}{\omega^2 - \omega_0^2 + 2\mathrm{i}\gamma\omega}\right]^{-1}. \tag{11}$$

To derive the effective permeability of this system we recognize that the component of the fields parallel to the yz-surface must be continuous. In the vacuum, this field is $H_{app} = \mu_0^{-1}B_{app}$ and, in the medium, $H = (\mu\mu_0)^{-1}B$. Hence,

$$\mu_0^{-1}B_{app} = H_{app} = (\mu\mu_0)^{-1}B. \tag{12}$$

Comparing with (11) gives

$$\mu = \left[1 + \frac{2\mu_0 I^2 m^{-1}a^{-2}}{\omega^2 - \omega_0^2 + 2\mathrm{i}\gamma\omega}\right]^{-1}. \tag{13}$$

Provided that the damping is small, negative values of μ are obtained in the range

$$\sqrt{\omega_0^2 - \frac{2\mu_0 I^2}{ma^2}} < \omega < \omega_0. \tag{14}$$

In order that we can neglect the effects of collisions between the moving wires, we take the wire radius $r \ll a$. In figure 2 we have plotted the effective permeability for the following values of the parameters:

$$a = 10^{-3}\ \mathrm{m}$$
$$m \simeq 2.8 \times 10^{-4}\ \mathrm{kg\ m^{-3}}$$
$$I \simeq 1.6\ \mathrm{A}.$$

These values correspond to copper wires (mass density: 8890 kg m^{-3}) of radius 10^{-4} m and carrying a current density $j = 5 \times 10^7$ A m^{-2}. We took ω_0, which depends on the tension exerted, to be $2\pi 50$ Hz and the damping rate $\gamma = 0.5$. In figure 2(d) the corresponding displacement of the wires in units of the spacing is shown for an applied field of 5×10^{-4} T.

Quite a different scenario pertains if the fields are sufficiently small that the displacement of the wires is small relative to their spacing. Since this avoids the possibility of collisions between the wires, this is in some ways the more interesting regime. We are also now free to allow for a larger wire radius relative to their spacing.

To derive the response we rewrite (7) as

$$m\ddot{u}_x = -\beta u_x - 2m\gamma\dot{u}_x - IB_{app}\exp(-\mathrm{i}\omega t). \tag{15}$$

We explain this new equation as follows: in the absence of B_{app}, a small displacement of the wires relative to one another finds the wires in a local potential minimum due partly to the restoring force $-\alpha u_x$ created by the tension in the wires and seen in equation (7), but also assisted by the fact that the currents are arranged in a stable local minimum created by the repulsion of opposite currents. This combined restoring force is written as $-\beta u_x$. Long-range forces do not set in until $u_x > a$. Now we calculate

$$u_x = \frac{Im^{-1}}{\omega^2 - {\omega_0'}^2 + 2\mathrm{i}\gamma\omega} B_{app}\exp(-\mathrm{i}\omega t) \tag{16}$$

where

$${\omega_0'}^2 = \beta/m. \tag{17}$$

Although the wires see only their local minimum, the sample as a whole experiences a net induced magnetization:

$$B_{ind}\exp(-\mathrm{i}\omega t) = -2\mu_0 I a^{-2} u_x = \frac{-2\mu_0 I^2 m^{-1} a^{-2}}{\omega^2 - {\omega_0'}^2 + 2\mathrm{i}\gamma\omega} B_{app}\exp(-\mathrm{i}\omega t). \tag{18}$$

Hence we calculate

$$B = \mu_s B_{app} = B_{app} + B_{ind} = \left[1 - \frac{2\mu_0 I^2 m^{-1} a^{-2}}{\omega^2 - {\omega_0'}^2 + 2\mathrm{i}\gamma\omega}\right] B_{app} \tag{19}$$

and identify

$$\mu_s = 1 - \frac{2\mu_0 I^2 m^{-1} a^{-2}}{\omega^2 - {\omega_0'}^2 + 2\mathrm{i}\gamma\omega}. \tag{20}$$

In contrast to the large-field case, μ_s diverges at the resonance frequency and is zero at

$$\omega = \sqrt{{\omega_0'}^2 + \frac{2\mu_0 I^2}{ma^2}}. \tag{21}$$

For comparison with our results for the large-field case we assume copper wires with $\omega_0 = \omega_0'$ as before and the same values of a and γ. The remaining parameters that we assume are the current density, $j = 10^7$ A m^{-2}, and wire radius, $r = 2.5 \times 10^{-4}$ m, in which case

$$m \simeq 1.8 \times 10^{-3}\ \mathrm{kg\ m^{-3}} \qquad I \simeq 2.0\ \mathrm{A}.$$

The effective permeability is plotted in figure 3 and the displacement is plotted in figure 3(d) for an applied field of 10^{-4} T: roughly twice that of the Earth.

The assumption of small displacements of the wires allows us to consider more complex structures. A single set of wires parallel to the z-axis only produces magnetic activity in the xy-plane, but introducing a second set of wires parallel to the y-axis gives magnetic activity

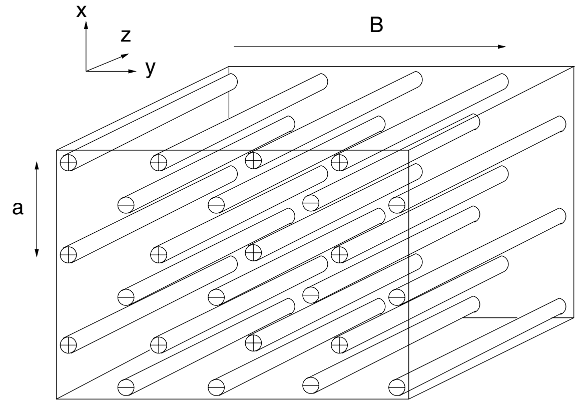

Figure 1. An array of current-carrying wires designed to simulate a magnetic plasma. The wires move in response to external magnetic fields and thus produce a magnetic polarization of the sample. For the purposes of the calculation the wires are assumed long compared to the depth in the x-direction, and the sample is also assumed to be long in the y-direction.

along all three axes. Obviously the second set of wires has no effect as the field lies parallel to the second set of wires, so we can immediately write for the new wire grid structure

$$\mu_{2y} = \mu_{2z} = \mu_s = 1 - \frac{2\mu_0 I^2 m^{-1} a^{-2}}{\omega^2 - {\omega'_0}^2 + 2\mathrm{i}\gamma\omega}. \tag{22}$$

On the other hand, a field parallel to the x-axis moves both sets of wires producing twice the polarization:

$$\mu_{2x} = 1 - \frac{4\mu_0 I^2 m^{-1} a^{-2}}{\omega^2 - {\omega'_0}^2 + 2\mathrm{i}\gamma\omega}. \tag{23}$$

Figures 2 and 3 indicate that large values of the effective permeability are attainable using this prescription and for a wide range of strengths of the applied magnetic field. In deriving our results for the effective permeability we have neglected the finite length of the wires which must be long in comparison to their displacement in order that the derived permeability represents a good approximation in each case. We have also assumed that the applied magnetic field is uniform over the length of the wire. The fundamental resonance frequency of a narrow wire under tension of length l is given by

$$\omega_0 = \pi c/l \tag{24}$$

where $c = \sqrt{T/m}$ is the speed of transverse waves on the wire and T is the tension. In order for example to screen a uniform applied field of frequency ω, the tension should be adjusted such that this fundamental mode frequency lies near ω. Of course in practice any source of magnetic field will contain many spatial Fourier components. If we consider the geometry of figure 1, then for a component of the applied field which varies as $\exp(\mathrm{i}k_z z)$, the screening will only be effective provided that $k_z^{-1} \gg l$. For fields which are rapidly varying in space a series of short lengths of wire with a correspondingly reduced tension may therefore be appropriate.

In conclusion, we have presented a scheme for realizing a negative permeability at very low frequencies of a few hertz. The proposed structure consists of an array of current-carrying wires. Because the wires occupy a small fraction of the structures' volume these structures will be lightweight and the resonant frequency will be tunable through the tension exerted in the wires. Excitation of surface modes of the structure will be possible where $\mu = -1$ as

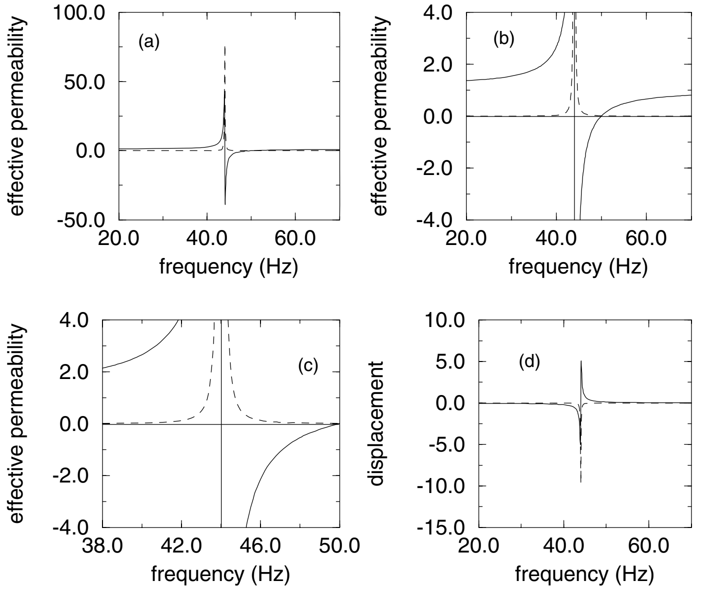

Figure 2. (a)–(c) The effective permeability as a function of frequency of the array of current-carrying wires for the case where the displacement is large compared to their spacing. The parameters are given in the text. (d) The displacement of the wires in units of their spacing for an applied field of 5×10^{-4} T. The solid curves are the real parts and the dashed curves the imaginary parts in each case.

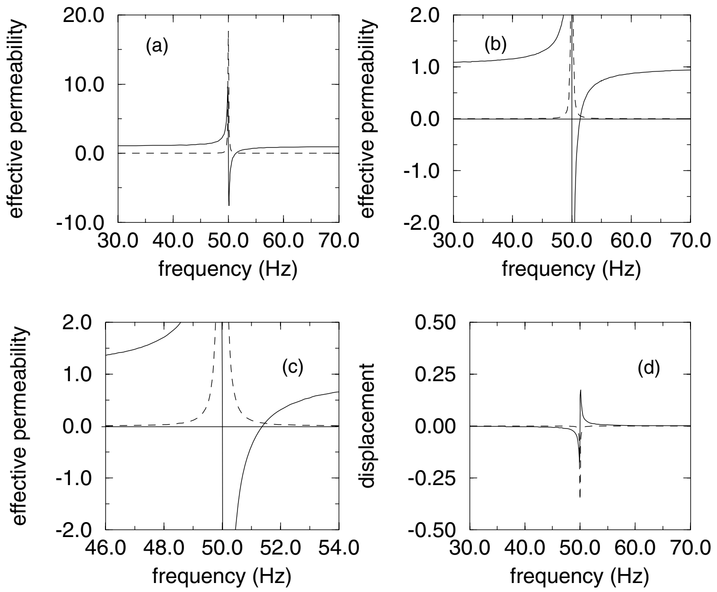

Figure 3. (a)–(c) The effective permeability as a function of frequency of the array of current-carrying wires for the case where the displacement is small compared to their spacing. The parameters are given in the text. (d) The displacement of the wires in units of the wire spacing for an applied field of 10^{-4} T. The solid curves are the real parts and the dashed curves the imaginary parts in each case.

well as a bulk magnetic plasmon mode where $\mu = 0$. The large positive permeability below the resonance frequency could find an application for the structure as a magnetic conductor in eliminating the mains signal from a sensitive piece of equipment. This could be achieved by forming a mesh of wires into a box and adjusting the tension as suggested in order that the fundamental resonance frequency of the wires was near 50 Hz. The magnetic fields would then be channelled though the sides of the box where the permeability is large thus shielding the interior.

References

[1] Veselago V G 1968 *Sov. Phys.–Usp.* **10** 509
[2] Smith D R, Padilla W J, Vier D C, Nemat-Nasser S C and Schultz S 2000 *Phys. Rev. Lett.* **84** 4184
[3] Pendry J B 2000 *Phys. Rev. Lett.* **85** 3966
[4] Pendry J B, Holden A J, Robins D J and Stewart W J 1999 *IEEE Trans. Microw. Theory Tech.* **47** 2075
[5] O'Brien S and Pendry J B 2002 *J. Phys.: Condens. Matter* **14** 6383
[6] O'Brien S and Pendry J B 2002 *J. Phys.: Condens. Matter* **14** 4035